ESTIMATES OF THE POSSIBILITY OF OBTAINING HIGH-ENERGY DENSITIES IN THE ELECTRICAL EXPLOSION OF CYLINDRICAL SHELLS

Yu. D. Bakulin and A. V. Luchinskii

UDC 533.9.07:537.529

Compression of a thin, metallic cylindrical shell at collapse rates of $\gtrsim 10$ cm/μsec is of interest from the viewpoint of a number of physical problems, including the problem of exciting a pulsed thermonuclear reaction. Such velocities correspond to energy densities in the shell of $\varepsilon \gtrsim 5$ MJ/g. Two means of introducing the energy exist in an electrodynamic method of shell compression are as follows: compression by a magnetic field and extraction of Joulean heat with subsequent transformation into kinetic energy. The process actually proceeds simultaneously by both means, although the leading role in the different stages of compression can be played by first one, then the other factor.

Let us estimate the possible shell dimensions. It follows from the requirement of uniformity of the motion that the initial shell radius r_0 should not exceed half its length significantly. If the ratio of thickness to r_0 is within the limits 0.01-0.1, then the total energy reserve in a copper shell is $(10\text{-}100)r_0^3$ for $\varepsilon \sim 5$ MJ/g. Let $\sim 20\%$ of the energy W_0 initially stored in the source be transferred successfully to the shell; then

$$r_0 \leqslant 0.1\sqrt[3]{(2-20)W_0}.$$

For $W_0 \lesssim 25$ MJ the shell radius should be $r_0 \leqslant 1$ cm. The time of energy absorption by the shell t_e should be less than its collapse. For a collapse rate of ~ 10 cm/μsec, this results in the need to assure $t_e \leqslant 0.1$ μsec.

Therefore, the problem exists of extracting an energy density of $\geqslant$ 5MJ/g in a shell of up to 2-cm size in a time $\leqslant 0.1$ μsec. Let us estimate the prospects of using different electrical engineering circuits for these purposes.

1. Estimates of the Shell Joulean Energy in Circuits with Lumped Inductance L and Capacitance C

Let us first examine the relationship between the time t_b and t_t, where t_b is the time from the beginning of current passage over the conductor to the time of the explosion and t_e is the characteristic time of explosion of the wire. It is known [1] that for an LC-loop (the loop resistance is $< \sqrt{L/C}$) the explosion occurs near the fourth period upon compliance with the condition*

$$S^2 = \frac{1}{\pi A} t_b I_e^2, \tag{1.1}$$

where I_e is the current at which the explosion starts; S, wire cross-sectional area; A, a constant equal to 2000 $MA^2 \cdot \mu sec/cm^3$ for copper, for example. It can be shown that (1.1) can be rewritten in the form

$$S^2 = \eta t_b I_e^2, \tag{1.2}$$

for a broad circle of dependences I(t), where η varies within the limits 10^{-4}—$10^{-3})/\pi$. In the case of an RC loop, when $R > \sqrt{L/C}$, the relationship

$$l^2 = 1.6 \cdot 10^6 t_b U_0^2$$

*The following units are used everywhere in the formulas: cm, g, μsec, MJ, MA, MV, Ω, μH, μF.

Chelyabinsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 115-122, January-February, 1980. Original article submitted February 10, 1979.

0021-8944/80/2101-0107$07.50

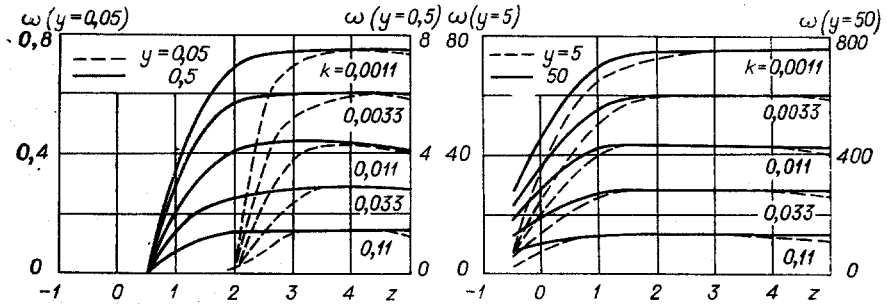

Fig. 1

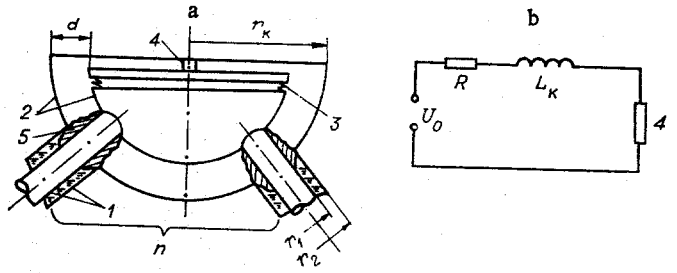

Fig. 2

is obtained analogously to (1.2). The fact that the time of the explosion is determined by the length rather than the section of the wire for a given initial voltage U_0 is of interest.

Experiments and computations [2] show that the average wire resistance within the time of the explosion t_e approximately equals

$$R_e \sim 300\, R_0, \tag{1.3}$$

where R_0 is the initial resistance. In fact, the resistance certainly varies during the explosion and depends on the wire and loop parameters. However, relationship (1.3), true in order of magnitude, can be used for estimates of t_e.

For the case of inductive storage, the Joulean energy absorbed by the wire can be written in the form

$$W_J = \frac{LI_e^2}{2} a, \quad \text{where } a = 1 - \exp\left(-\frac{2R_e}{L} t\right).$$

Denoting the Joulean energy density in the wire by ε_J and using (1.2) and (1.3), we have

$$t_e \approx -\frac{\ln(1-a)}{2a} 3 \cdot 10^4 \eta \varepsilon_J t_b.$$

As has already been said, $\eta \approx 10^{-4}$. The quantity $-\ln(1-a)/2a$ remains close to 1 in a broad range of variation of a. Therefore, the expression governing the relationship between t_e and t_b has the form

$$t_e \approx 3\varepsilon_J t_b. \tag{1.4}$$

For a RC loop the relationship similar to (1.4) is written in the form

$$t_e \approx 10^4 \varepsilon_J t_b. \tag{1.5}$$

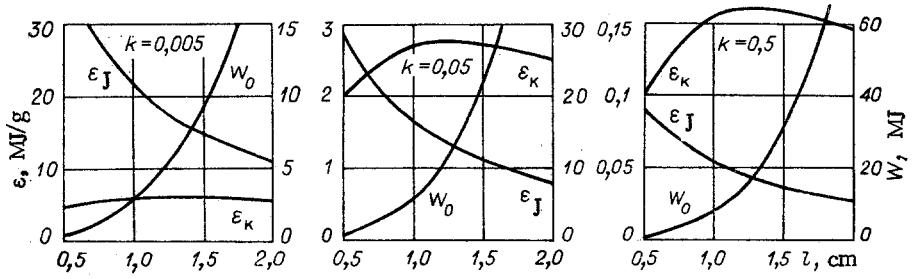

Fig. 3

It is seen from (1.4) and (1.5) that the ratio t_e/t_b will be greater, the greater the energy density in the wire obtained during the explosion. Sharpening the pulse ($t_e < t_b$) is possible for inductive storage only for $\varepsilon_J \leqslant 0.3$ MJ/g. In ordinary experiments on the electrical explosion of wires $\varepsilon_J \approx 0.015$ MJ/g, and $t_e/t_b \approx 0.05$. For the energy densities $\varepsilon_J \gtrsim 5$MJ/g interesting us, we should expect $t_e/t_b \geqslant 10$.

Let us now examine the question of the reality of obtaining the ε_J needed in an LC loop. If it is required that a significant fraction of the energy stored in the condensers up to the time of the explosion go to the inductance, then it follows from (1.4) that it is impossible to realize an explosion with the energy density needed within the first half-period Attempts to obtain the needed parameters of the explosion for a fluctuating current appears to be unreal primarily because a loop with an oscillation frequency of tens of megahertz for an energy reserve of more than a megajoule would be required for this. In addition, the fluctuating mode of the current passing through the plasma being dissipated should result in magnification of the instability of shell motion.

A circuit with a commutator, which first permits a slow current build-up in the inductance, and then switches it to the shell being exploded, is also not promising. According to (1.4), it would be required from such a commutator that it magnify its resistance more strongly than the shell itself in a time less than $t_b \leqslant 0.1\ t_e \sim 0.01$ μsec. Examination of an RC loop results in the same deduction. In this case, for the explosion to occur with a sufficient current, the current front should be commensurate with $t_b \leqslant 10^{-4} t_e \approx 10^{-5}$ μsec (see (1.5)).

Therefore, none of the circuits considered permits obtaining the needed Joulean energy density for the shell.

2. Estimates of the Shell Kinetic Energy in Circuits with Lumped L and C

Now, let us consider the possibility of obtaining the requisite shell kinetic energy density ε_k when it is compressed by a magnetic field in a loop with lumped L and C. We consider the shell thin and made from an incompressible material. Then the equation of shell motion is written in the form

$$r_b \frac{d^2 r_b}{dt^2} = -\frac{10^{-2} I^2}{\pi\gamma\left(r_0^2 - r_*^2\right)}, \tag{2.1}$$

where r_b is the outer shell radius, I, current through the shell; r_0 and r_*, initial values of the outer and inner shell radii; and γ, shell density.

The equation for the electrical loop has the form

$$\frac{1}{C}\int_0^t I dt + L\frac{dI}{dt} + \frac{d}{dt}(L_0 I) = U_0, \quad L_0 = 2\cdot 10^{-3} l \ln\frac{r_T}{r_b}, \tag{2.2}$$

where C and L are the capacitance and inductance of the loop (excluding the shell); L_0 and l, shell inductance and length; and r_T, radius of the reverse current conductor.

Let us introduce the dimensionless quantities

$$\Theta = t/\sqrt{2\cdot 10^{-3} lC}, \quad x = r_b/r_0, \quad J = I/223 r_0 \sqrt{\frac{\pi\gamma\left(r_0^2 - r_*^2\right)}{lC}},$$

$$k = \frac{r_0 - r_*}{r_0}, \quad z = (L + L_0)/2\cdot 10^{-3} l, \quad \omega = \frac{lC}{500 r_0^2}\left(\frac{dr_b}{dt}\right)^2.$$

The last quantity characterizes the dimensionless energy density. We obtain the system of equations

$$(z - \ln x)\frac{d^2J}{d\Theta^2} - \frac{2}{x}\frac{dx}{d\Theta}\frac{dJ}{d\Theta} + \left[1 + \frac{1}{x^2}\left(\frac{dx}{d\Theta}\right)^2 - \frac{1}{x}\frac{d^2x}{d\Theta^2}\right] J = 0, \tag{2.3}$$

$$x\frac{d^2x}{d\Theta^2} = -J^2, \quad \omega = \left(\frac{dx}{d\Theta}\right)^2$$

from (2.1) and (2.2). The initial conditions for this system are written for $\Theta = 0$ in the form $x = 1$, $J = 0$, $dx/d\Theta = 0$, $dJ/d\Theta = y_0$. Starting from the fact that for $t = 0$

$$dI/dt = U_0/(L + L_0),$$

we obtain

$$y_0 = \frac{5.7\cdot 10^{-2}}{\sqrt{\gamma\left(r_0^2 - r_*^2\right)}}\frac{U_0 C}{z r_0}.$$

System (2.3) was solved on an electronic computer. The count was made to $x = \sqrt{k(2-k)}$ (shell total compression). The shell energy head after the first half-period is of no interest, consequently, the count is cut off for $J = 0$. The parameters z, k, y were varied in the computations, and the maximum of $\omega(\Theta)$ was determined. The dependences of ω on z are presented in Fig. 1 for different k and y_0. For $zy > 1000$ these dependences are approximated with an error $< 10\%$ by the formula

$$\omega = 6.4y \lg (0.24/k).$$

Returning to the dimensional variables, we obtain that upon conserving the condition

$$\frac{U_0 C}{r_0^2} \gtrsim 5\cdot 10^4 \sqrt{k(2-k)} \tag{2.4}$$

the energy density will equal

$$\varepsilon_k \approx \frac{6\cdot 10^{-3} U_0 \lg \frac{0.24}{k}}{L_* \sqrt{k(2-k)}}, \tag{2.5}$$

where $L_* = L + 2\cdot 10^{-3}\, l \ln(r_T/r_0)$ is the total loop induction at the initial time. If condition (2.4) is not satisfied, then ε_k turns out to be less than the quantity defined by (2.5).

It follows from (2.5) that if $k > 10^{-3}$, then $\varepsilon_k < 0.5 U_0/L_k$ and to obtain $\varepsilon_k \gtrsim 5$ MJ/g the initial derivative of the current should be greater than 10 MA/μsec. Let us note that the requirement (2.4) for such a derivative reduces to $W_0 \gtrsim 10^4 L_* r_0^2$. If $r_0 \sim 1$ cm and $W_0 \leqslant 10$ MJ/g, then the initial loop inductance should be $L_* \leqslant 10^{-3}$ μH. The production of capacitive batteries with such parameters is scarcely possible.

Therefore, circuits with lumped L and C cannot apparently assure the necessary shell energy density of both Joulean and that obtained because of acceleration by a magnetic field.

3. Estimate of the Prospects of Using Lines with Distributed Parameters

Let us consider the circuit displayed in Fig. 2a. Similarly to [3], n parallel lines 1 are connected to the load 4 by this circuit. In our case, this connection is made through a collector 2 represented in the form of two concentric hemispheres, r_k, radius of the outer hemisphere; and d, gap magnitude. A discharger 3 is placed at the discontinuity of the inner hemisphere. Each line is two coaxial tubes with inner r_1 and outer r_2 radii and length l_0. The gap between the tubes is filled with a dielectric with the permittivity ψ. The gap be-

tween the collector hemispheres is separated by the insulators 5 from the gaps between the line tubes and is filled with a dielectric with permittivity ∿1.

If the time of electromagnetic wave passage in the collector $t_k \sim 8.6\cdot10^{-5} r_k$ is much less than the time of the process under consideration, then for the time $< 6\cdot10^{-5} l_0\sqrt{\psi}$ the equivalent circuit of the apparatus can be represented in the form shown in Fig. 2b. The internal source resistance is $R = \frac{60}{n\sqrt{\psi}} \ln\frac{r_2}{r_1}$, and the collector inductance approximately equals

$$L_k \approx 2\cdot10^{-3}\frac{r_k}{2}\ln\frac{r_k}{r_k - d}.$$

The collector gap should sustain the voltage U_0. We shall consider a mean electric field intensity ∿ 1.5 MV/cm allowable in the collector insulator; then $d = 0.6\ U_0$. In order for the gap in the lines to sustain the voltage U_0 it is necessary to have $r_2 \geqslant r_1 \exp(U_0/r_1E_0)$, where E_0 is the allowable field intensity in the substance filling the line. The quantity of lines is hence determined by the collector area (we shall later consider the lines to fill 0.75 of the collector area).

Let the time of the process be T and $t_k \sim 0.1$ T, then we obtain the following expression for the system wave resistance:

$$R(r_1) = \frac{2.9\cdot10^{-5}U_0 r_1}{E_0 T^2\sqrt{\psi}} \exp\left(\frac{2U_0}{r_1E_0}\right).$$

The minimal value of the resistance has the form

$$\min R(r_1) = \frac{1.6\cdot10^{-4}U_0^2}{E_0^2T^2\sqrt{\psi}} \quad \text{for} \quad r_1 = 2\frac{U_0}{E_0}, \quad r_2 = 1.65r_1.$$

Let us take $l_0 = 1.5\cdot10^4 T/\sqrt{\psi}$. We shall consider the lines filled with water with $\psi = 80$ and $E_0 = 0.3$ MV/cm. Then we obtain the following system of relationships

$$\begin{aligned} &l_0 = 1.7\cdot10^3 T,\ r_k = 1200\ T,\\ &r_1 = 6.7\ U_0,\ L_k = -1,\ 2\ T\ln(1 - 5\cdot10^{-4}U_0/T),\\ &r_2 = 11\ U_0,\ R = 2\cdot10^{-4}(U_0/T)^2,\\ &n = 2.2\cdot10^4(T/U_0)^2,\ W_0 = 1.2\cdot10^3T^3. \end{aligned} \tag{3.1}$$

to select the apparatus dimensions for given T and U_0, where W_0 is the electric energy stored in the lines.

As an illustration, let us present an example in which T = 0.2 μsec and U_0 = 2 MV. On the basis of (3.1), the apparatus should be 200 parallel lines with the dimensions $r_1 \approx 13$ cm, $r_2 \approx 22$ cm, and $l_0 \approx 350$ cm. The lines are connected to a collector with $r_k \approx 250$ cm and $L_k \approx 0.0012$ μH. The energy stored in the lines is $W_0 \approx 10$ MJ, and the wave resistance is $R \approx 0.02\ \Omega$.

Let us consider Joulean heating and magnetic field compression of the shell included in an apparatus of such kind. We consider the shell incompressible, as before, but to have a finite (varying with time) thickness. The equation of motion for the centers of mass of the sectorial shell elements is written similarly to (2.1). Let us introduce the variable

$$x = \sqrt{1 - \frac{r_0^2 - r_*^2}{r_b^2}}. \tag{3.2}$$

Because of the incompressibility of the shell material, its inner radius is $r_e = xr_b$. For the outer radius the equation of motion is rewritten in the form

$$r_b\frac{d^2r_b}{dt^2} = (1-x)\left[\frac{1}{x}\left(\frac{dr_b}{dt}\right)^2 - \frac{2.7\cdot10^{-4}(1+x)^2I^2}{(r_0^2 - r_*^2)(1-x^3)}\right]. \tag{3.3}$$

The specific kinetic energy of the shell equals

$$\varepsilon_k = 0.2\left(\frac{1}{1+x}\frac{dr_b}{dt}\right)^2. \tag{3.4}$$

In connection with (1.4), the shell resistance can be considered constant

$$R_e = 1.7\cdot 10^{-4}\frac{l}{r_0^2 - r_*^2}. \tag{3.5}$$

The shell inductance is comprised of the outer L_b and inner L_e. If it is considered that L_b is determined by the magnetic field in the space between $r_b(t)$ and $r = l$, while the rest of the inductance is in L_k, then

$$L_b = 2\cdot 10^{-3}\, l \ln (l/r_b). \tag{3.6}$$

In determining L_e we shall consider the current distributed uniformly over the shell. Let us note that such a consideration is justified by the fact that $t_b \ll t_e$, and therefore, the shell conductivity is low and the skin layer conductivity is large during the whole process under consideration. Then for the internal inductance we obtain

$$L_e = 10^{-3} l\left(1 + \frac{2x^2}{1-x^2}\ln x\right). \tag{3.7}$$

Taking (3.6) and (3.7) into account, we write the equation of the loop

$$\frac{dI}{dt} = \frac{U_0 - \left(\frac{1.67\cdot 10^{-4} l}{r_0^2 - r_*^2} + \frac{4\cdot 10^{-3} l}{r_b(1-x^2)}\frac{dr_k}{dt}\ln x + R\right) I}{L_b + 10^{-3} l\left(1 + \frac{2x^2}{1-x^2}\ln x + 2\ln\frac{l}{r_b}\right)}. \tag{3.8}$$

We obtain the following equation for the specific Joulean energy

$$\frac{d\varepsilon_J}{dt} = 5.6\cdot 10^{-6}\left(\frac{I}{r_0^2 - r_*^2}\right)^2. \tag{3.9}$$

Equations (3.3), (3.8), and (3.9) together with the relations (3.2) and (3.4) yield a complete system coverning the processes in the electrical loop, the shell motion, and the set of Joulean and kinetic energies of the shell. The initial conditions for this system are written in the form $r_b = r_0$, $I = 0$, $\varepsilon_J = 0$ at $t = 0$.

The system was computed on an electronic computer. The computations were performed up to the time of total compression of the shell, i.e., to t_c, for which $r_b = \sqrt{r_0^2 - r_*^2}$. By iteration, the parameter T in (3.1) was selected in such a way that the t_c obtained in the computations equaled T. The parameters k, l, r_0 and U_0 were variated. The dependences of the Joulean ε_J and kinetic ε_k energy densities obtained as a result of the computations described are presented in Fig. 3, as is also that for the required energy store in the lines W_0 on the shell length l.

The results of such computations are in the nature of very approximate estimates and cannot pretend to any accuracy; however, they apparently yield a number of qualitative regularities. The abrupt drop in the maximum energy density as the relative shell thickness increases is evident. Thus, an increase in k from 0.005 to 0.1 reduces the maximum ε_k approximately 4 times. The voltage to which the lines are charged affects the energy density much more weakly. The computations did not disclose any essential dependence of ε_J on U_0. The quantity ε_k has a weak maximum in the area of $U_0 = 2$ MV. As the shell length increases, ε_J drops but ε_k grows slightly or has a maximum at medium lengths $l \sim l$ cm. Naturally the energy store required grows with the increase in l. Upon conserving the length while the radius increases, ε_J drops sharply but ε_k grows. Let us note that, in general, the less the shell thickness, the greater the fraction of energy density in ε_J. It is seen from Fig. 3 that the values of the energy density attain several megajoules per gram for small k.

The considerably greater prospects for applying systems of lines with distributed parameters as compared with the other schemes considered can be considered the general deduction from the estimates presented.

LITERATURE CITED

1. G. Anderson and F. Neilson, "Application of the action integral concept in investigations of exploding wires," in: Exploding Wires [Russian translation], IL Moscow (1963).
2. Yu. D. Bakulin, V. F. Kuropatenko, and A. V. Luchinskii, "Magnetohydrodynamic analysis of exploding conductors," Zh. Tekh. Fiz., 46, No. 9 (1976).
3. K. R. Prestwich, "Harp, a short pulse, high current electron beam accelerator," IEEE Trans. Nucl. Sci., NS-22, No. 3 (1975).

EQUATION OF STATE OF HYDROGEN UP TO 10 MBAR

V. P. Kopyshev and V. V. Khrustalev

UDC 536.71

The problem of calculating the equation of state of hydrogen in its exact formulation is insoluble by contemporary methods, and therefore it is necessary to resort to physical models or formal interpolations. One such model, the compressible covolume model (CCM), was formulated in [1] under another name.

The "covolume" V° (a function of the pressure p) is a synonym for the "elastic volume" of [1]. In the present article the CCM is generalized to the case of a nonideal plasma, the covolumes of the molecular and atomic phases of hydrogen are constructed mainly on the basis of experimental data, and the complete equation of state of hydrogen up to a pressure of 10 Mbar is calculated in the temperature range T above 100°K for $p < 10$ kbar, and above the Debye temperature $\Theta(p)$ for $p > 10$ kbar.

Assuming the results and notation of [1], we note here only the generalization to the case of a plasma. There are five kinds of particles, differing in the index n: molecules H_2 ($n = m$), atoms H ($n = a$), ions H_2^+ ($n = i$), protons H^+ ($n = p$), and electrons e ($n = e$); the α_n are the concentrations of the particles, and the $V_n^\circ(p)$ are the covolumes. For molecules and atoms the covolumes are identified with the zero isotherms ($T = 0$) of the corresponding phases. According to estimates in [2] the Coulomb field of the charged particles is strongly self-screened, and can be neglected.

Electrons are formed as a result of the ionization of atoms or molecules. Strictly speaking, the very idea of ionization becomes indefinite when the particles are "close"; rather, one should speak of the excitation of electrons, of the removal of their degeneracy. On the other hand, when the temperature is lowered, the degeneracy of excited electrons shows up in the fact that they "sit" in orbits around protons or ions, forming neutral atoms or molecules. In accord with the fundamental idea of the additivity of free and elastic volumes [1], we add to the free volume $\alpha_p RT/p$ of electrons formed in the $H = H^+ + e$ reaction the elastic volume $\alpha_p V_a^\circ(p)$. At high temperatures there is no degeneracy, and only the first term is important; at low temperatures total degeneracy is approached, and only the second term is important. Similarly, for the $H_2 = H_2^+ + e$ reaction we add $\alpha_i V_m^\circ(p)$ to $\alpha_i RT/p$. From the law of conservation of charge $\alpha_p + \alpha_i = \alpha_e$.

We assume that in the equations from [1] an ideal electron gas (ionized or excited) has a Maxwell–Boltzmann distribution. Then the form of the CCM equations remains unchanged for a plasma if we formally supplement the definitions of the covolumes of charged particles: $V_p^\circ = V_a^\circ$, $V_i^\circ = V_m^\circ$, $V_e^\circ = 0$.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 122-128, January-February, 1980. Original article submitted October 16, 1978.